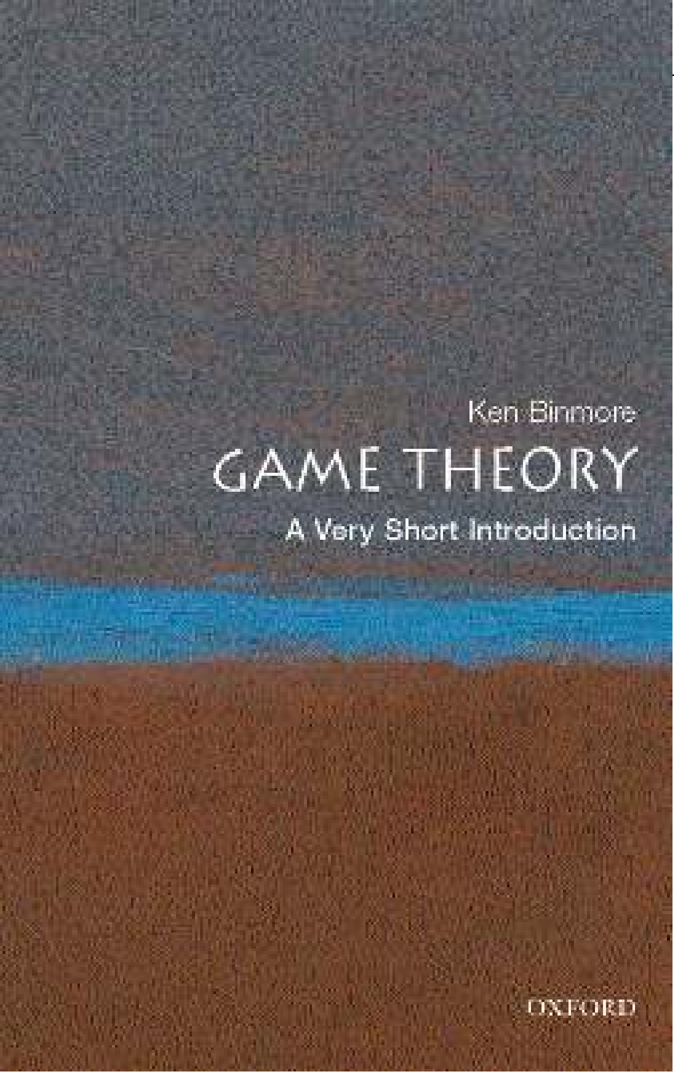
Ken Binmore
GAME THEORY
A Very Short Introduction
OXFORD

Game Theory: A Very Short Introduction

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To
Peter and Nina

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List of illustrations

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Chapter 1

The name of the game

What is game theory about?

When my wife was away for the day at a pleasant little conference in Tuscany, three young women invited me to share their table for lunch. As I sat down, one of them said in a sultry voice, 'Teach us how to play the game of love', but it turned out that all they wanted was advice on how to manage Italian boyfriends. I still think they were wrong to reject my strategic recommendations, but they were right on the nail in taking for granted that courting is one of the many different kinds of game we play in real life.

Drivers manoeuvring in heavy traffic are playing a driving game. Bargain-hunters bidding on eBay are playing an auctioning game. A firm and a union negotiating next year's wage are playing a bargaining game. When opposing candidates choose their platform in an election, they are playing a political game. The owner of a grocery store deciding today's price for corn flakes is playing an economic game. In brief, a game is being played whenever human beings interact.

Antony and Cleopatra played the courting game on a grand scale. Bill Gates made himself immensely rich by playing the computer software game. Adolf Hitler and Josef Stalin played a game that killed off a substantial fraction of the world's population. Kruschev and Kennedy played a game during the Cuban missile crisis that might have wiped us out altogether.

With such a wide field of application, game theory would be a universal panacea if it could always predict how people will play the many games of which social life largely consists. But game theory isn't able to solve all of the world's problems, because it only works when people play games *rationally*. So it can't predict the behaviour of love-sick teenagers like Romeo or Juliet, or madmen like Hitler or Stalin. However, people don't always behave irrationally, and so it isn't a waste of time to study what happens when people put on their thinking caps. Most of us at least try to spend our money sensibly – and we don't do too badly much of the time or economic theory wouldn't work at all.

Even when people haven't thought everything out in advance, it doesn't follow that they are necessarily behaving irrationally. Game theory has had some notable successes in explaining the behaviour of spiders and fish, neither of which can be said to think at all. Such mindless animals end up behaving as though they were rational, because rivals whose genes programmed them to behave irrationally are now extinct. Similarly, companies aren't always run by great intellects, but the market is often just as ruthless as Nature in eliminating the unfit from the scene.

Does game theory work?

In spite of its theoretical successes, practical men of business used to dismiss game theory as just one more ineffectual branch of social science, but they changed their minds more or less overnight after the American government decided to auction off the right to use various radio frequencies for use with cellular telephones.

With no established experts to get in the way, the advice of game theorists proved decisive in determining the design of the rules of the auctioning games that were used. The result was that the

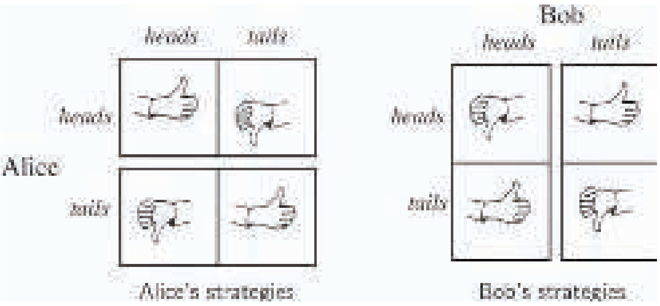
American taxpayer made a profit of $20 billion – more than twice the orthodox prediction. Even more was made in a later British telecom auction for which I was responsible. We made a total of $35 billion in just one auction. In consequence, *Newsweek* magazine described me as the ruthless, Poker-playing economist who destroyed the telecom industry!

As it turned out, the telecom industry wasn't destroyed. Nor is it at all ruthless to make the fat cats of the telecom industry pay for their licences what they think they are worth – especially when the money is spent on hospitals for those who can't afford private medical care. As for Poker, it is at least 20 years since I played for more than nickels and dimes. The only thing that *Newsweek* got right is that game theory really does work when applied by people who know what they are doing. It works not just in economics, but also in evolutionary biology and political science. In my recent book *Natural Justice*, I even outrage orthodox moral philosophers by using game theory when talking about ethics.

Toy games

Each new big-money telecom auction needs to be tailored to the circumstances in which it is going to be run. One can't just take a design off the shelf, as the American government found when it hired Sotheby's to auction off a bunch of satellite transponders. But nor can one capture all the complicated ins and outs of a new telecom market in a mathematical model. Designing a telecom auction is therefore as much an art as a science. One extrapolates from simple models chosen to mimic what seem to be the essential strategic features of a problem.

I try to do the same in this book, which therefore contains no algebra and a minimum of technical jargon. It looks only at toy games, leaving aside all the bells and whistles with which they are complicated in real life. However, most people find that even toy games give them plenty to think about.



1. Alice and Bob's decision problem in Matching Pennies

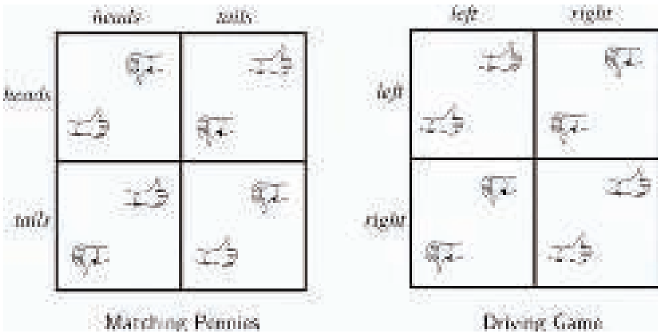
Conflict and cooperation

Most of the games in this book have only two players, called Alice and Bob. The first game they will play is Matching Pennies.

Sherlock Holmes and the evil Professor Moriarty played Matching Pennies on the way to their final confrontation at the Reichenbach Falls. Holmes had to decide at which station to get off a train. Moriarty had to decide at which station to lie in wait. A real-life counterpart is played by dishonest accountants and their auditors. The former decide when to cheat and the latter decide when to inspect the books.

In our toy version, Alice and Bob each show a coin. Alice wins if both coins show the same face. Bob wins if they show different faces. Alice and Bob therefore each have two strategies, *heads* and *tails*. Figure 1 shows who wins and loses for all possible strategy combinations. These outcomes are the players' *payoffs* in the game. The thumbs-up and thumbs-down icons have been used to emphasize that payoffs needn't be measured in money.

Figure 2 shows how all the information in Figure 1 can be assembled into a payoff table, with Alice's payoff in the southwest corner of each cell, and Bob's in the northeast corner. It also shows a two-player version of the very different Driving Game that we



2. Payoff tables. Alice chooses a row and Bob chooses a column

play every morning when we get into our cars to drive to work. Alice and Bob again have two pure strategies, *left* and *right*, but now the players' payoffs are totally aligned instead of being diametrically opposed. When journalists talk about a win-win situation, they have something like the Driving Game in mind.

Von Neumann

The first result in game theory was John Von Neumann's minimax theorem, which applies only to games like Matching Pennies in which the players are modelled as implacable enemies. One sometimes still reads dismissive commentaries on game theory in which Von Neumann is caricatured as the archetypal cold warrior – the original for Dr Strangelove in the well known movie. We are then told that only a crazed military strategist would think of applying game theory in real life, because only a madman or a cyborg would make the mistake of supposing that the world is a game of pure conflict.

Von Neumann was an all-round genius. Inventing game theory was just a sideline for him. It is true that he was a hawk in the Cold War, but far from being a mad cyborg, he was a genial soul, who liked to party and have a good time. Just like you and me, he preferred cooperation to conflict, but he also understood that the way to achieve cooperation isn't to pretend that people can't sometimes profit by causing trouble.

Cooperation and conflict are two sides of the same coin, neither of which can be understood properly without taking account of the other. To consider a game of pure conflict like Matching Pennies isn't to claim that all human interaction is competitive. Nor is one claiming that all human interaction is cooperative when one looks at a game of pure coordination like the Driving Game. One is simply distinguishing two different aspects of human behaviour so that they can be studied one at a time.

Revealed preference

To cope with cooperation and conflict together, we need a better way of describing the motivation of the players than simply saying that they like winning and dislike losing. For this purpose, economists have invented the idea of *utility*, which allows each player to assign a numerical value to each possible outcome of a game.

In business, the bottom line is commonly profit, but economists know that human beings often have more complex aims than simply making as much money as they can. So we can't identify utility with money. A naive response is to substitute happiness for money. But what is happiness? How do we measure it?

It is unfortunate that the word 'utility' is linked historically with Victorian utilitarians like Jeremy Bentham and John Stuart Mill, because modern economists don't follow them in identifying utility with how much pleasure or how little pain a person may feel. The modern theory abandons any attempt to explain how people behave in terms of what is going on inside their heads. On the contrary, it makes a virtue of making no psychological assumptions at all.

We don't try to explain *why* Alice or Bob behave as they do. Instead of an explanatory theory, we have to be content with a descriptive theory, which can do no more than say that Alice or Bob will be acting inconsistently if they did such-and-such in the past, but now plan to do so-and-so in the future. In game theory, the object is to observe the decisions that Alice and Bob make (or would make) when they aren't interacting with each other or anyone else, and to deduce how they will behave when interacting in a game.

We therefore don't argue that some preferences are more rational than others. We follow the great philosopher David Hume in regarding reason as the 'slave of the passions'. As he extravagantly remarked, there would be nothing *irrational* about his preferring the destruction of the entire universe to scratching his finger. However, we go even further down this road by regarding reason purely as an instrument for avoiding inconsistent behaviour. Any consistent behaviour therefore counts as rational.

With some mild assumptions, acting consistently can be shown to be the same as behaving as though seeking to maximize the value of something. Whatever this abstract something may be in a particular context, economists call it utility. It needn't correlate with money, but it sadly often does.

Taking risks

In acting consistently, Alice may not be aware that she is behaving as though maximizing something we choose to call her utility. But if we want to predict her behaviour, we need to be able to measure her utility on a utility scale, much as temperature is measured on a thermometer. Just as the units on a thermometer are called degrees, we can then say that a *util* is a unit on Alice's utility scale.

The orthodoxy in economics used to be that such cardinal utility scales are intrinsically nonsensical, but Von Neumann fortunately didn't know this when Oskar Morgenstern turned up at his house one day complaining that they didn't have a proper basis for the numerical payoffs in the book on game theory they were writing together. So Von Neumann invented a theory on the spot that measures how much Alice wants something by the size of the risk she is willing to take to get it. We can then figure out what choice she will make in risky situations by finding the option that will give her the highest utility on average.

It is easy to use Von Neumann's theory to find how much utility to assign to anything Alice may need to evaluate. For example, how many utils should Alice assign to getting a date with Bob?

We first need to decide what utility scale to use. For this purpose, pick two outcomes that are respectively better and worse than any other outcome Alice is likely to encounter. These outcomes will correspond to the boiling and freezing points of water used to calibrate a Celsius thermometer, in that the utility scale to be constructed will assign 0 utils to the worst outcome, and 100 utils to the best outcome. Next consider a bunch of (free) lottery tickets in which the only prizes are either the best outcome or the worst outcome.

When we offer Alice lottery tickets with higher and higher probabilities of getting the best outcome as an alternative to a date with Bob, she will eventually switch from saying *no* to saying *yes*. If the probability of the best outcome on the lottery ticket that makes her switch is 75%, then Von Neumann's theory says that a date with Bob is worth 75 utils to her. Each extra percentage point added to her indifference probability therefore corresponds to one extra util.

When some people evaluate sums of money using this method, they always assign the same number of utils to each extra dollar. We call such people risk neutral. Those who assign fewer utils to each extra dollar than the one that went before are called risk averse.

Insurance

Alice is thinking of accepting an offer from Bob to insure her Beverley Hills mansion against fire. If she refuses his offer, she faces a lottery in which she ends up with her house plus the insurance premium if her house doesn't burn down, and with only the premium if it does. This has to be compared with her ending up for sure with the value of the house less the premium if she accepts Bob's offer.

If it is rational for Bob to make the offer and for Alice to accept, he must think that the lottery is better than breaking even for sure, and she must have the opposing preference. The existence of the insurance industry therefore confirms not only that it can be rational to gamble – provided that the risks you take are calculated risks – but that rational people can have different attitudes to taking risks. In the insurance industry, the insurers are close to being risk neutral and the insurees are risk averse to varying degrees.

Notice that economists regard the degree of risk aversion that a person reveals as a matter of personal preference. Just as Alice may or may not prefer chocolate ice-cream to vanilla, so she may or may not prefer to spend $1,000 on insuring her house. Some philosophers – notably John Rawls – insist that it is *rational* to be risk averse when defending whatever alternative to maximizing average utility they prefer, but such appeals miss the point that the players' attitudes to taking risks have already been taken into account when using Von Neumann's method to assign utilities to each outcome.

Economists make a different mistake when they attribute risk aversion to a dislike of the act of gambling. Von Neumann's theory only makes sense when the players are entirely neutral to the actual act of gambling. Like a Presbyterian minister insuring his house, they don't gamble because they enjoy gambling – they gamble only when they judge that the odds are in their favour.